CONVERGENCE OF TWO IMPLICIT NUMERICAL SCHEMES FOR DIFFUSION MATHEMATICAL MODELS WITH DELAY

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Abstract

Diffusion equations with delay are basic mathematical models in areas where time lags or hereditary characteristics are present, as in the dynamics of structured populations, heat transfer in materials with thermal memory or control theory.

In this work two implicit numerical difference schemes for the diffusion equations with delay

$$u_t(t,x) = a^2 u_{xx}(t,x) + a^2 c^2 u_{xx}(t-\tau,x),\ t > \tau,\ 0 \leq x \leq l,$$

with initial condition

$$u\left(t,x\right) = \phi\left(t,x\right),\ 0 \leq t \leq \tau,\ 0 \leq x \leq l,$$

and Dirichlet boundary conditions

$$u\left(t,0\right) = u\left(t,l\right) = 0,\ t \geq 0,$$

where a and $0 < c < 1$ are real constants, are proposed.

Using Taylor polynomial approximations to the derivatives of $u\left(t,x\right)$, implicit difference schemes are formulated as generalizations of the classic Crank-Nicolson and Richtmyer difference schemes for the diffusion equation.

The unconditional asymptotic stability of the new schemes is proved by analyzing the eigenvalues of the companion scheme matrix. Consistence, convergence and some properties of stability for these schemes are studied. For each one of the two schemes, an explicit general solution, depending on a complete set of co-solutions for the polynomial matrix equation associated to the scheme, are obtained. Illustrative examples of numerical results are also included.

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