Numerical mode matching for sound propagation in silencers with granular material

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ABSTRACT

1. Introduction.

Multidimensional methods are widely used for the acoustic modelling of automotive exhaust silencers [1]. Accurate predictions of the sound attenuation performance at mid and high frequencies require the consideration of three dimensional waves corresponding to higher order modes. Numerical approaches such as the boundary element method (BEM) and the finite element method (FEM) are usual design tools, although the associated computational expenditure of these fully numerical schemes can be considerable as the number of degrees of freedom increases [2, 3]. Thus, an effort has been made in the last two decades with a view to develop alternative modelling techniques that provide improved computational efficiency without sacrificing accuracy. Some of these techniques are based on hybrid approaches [4-11] that combine analytical and numerical aspects of the wave propagation. For example, in silencers with irregular but axially uniform cross section, a numerical approach can be used to model the transversal governing eigenequation [7]. The complete solution of the acoustic field in a particular silencer subdomain is obtained by adding the contribution of the axial propagating terms analytically. Finally, the acoustic coupling of all the subdomains involved is achieved through enforcing suitable compatibility conditions of acoustic pressure and axial velocity across the geometrical discontinuities. Bibliography tends to favour the point collocation technique and mode matching method as techniques to enforce these conditions [4, 11-13]. In general, for small dimensions mode matching has been shown to have some advantages in terms of speed and accuracy, due in part to symmetry properties, orthogonality of the transversal modes, the sensitivity of point collocation to the grid chosen and acoustic scattering at particular locations [4, 12, 13].

On the other hand, absorbent granular materials are studied here, from an acoustical point of view, as a potential alternative to the traditional fibrous materials used in dissipative silencers. As shown in earlier studies, sound propagation in granular materials can be modelled through complex and frequency dependent density and speed of sound [14]. Their acoustic properties can be predicted through models available in the bibliography, depending on [15, 16], which will be used in the context of the investigation.

In this work, a mathematical approach based on a numerical version of the mode matching method [8, 9] is presented to compute the transmission loss of silencers with granular material. Multidimensional sound propagation is taken into account in configurations with arbitrary, but axially uniform, cross section. Transversal material heterogeneities are included in the model. Also, the possibility of using different filling levels of granular material gives rise to cross sections with an abrupt change of properties and the existence of a transition between air and material involving a remarkable change in porosity (see Figure 1). The computational requirements of a full numerical scheme such as FEM are reduced through a method that combines analytical axial propagation terms with numerical transversal eigensolutions of the silencer heterogeneous cross section. Numerical mode matching [8, 9] is then used to couple the modal expansions associated with each silencer component and to obtain the complete solution of the wave equation. To this end, the compatibility conditions of the acoustic fields (pressure and axial acoustic velocity) at the geometric discontinuities between the silencer chamber and the inlet and outlet pipes are taken into account. Transmission loss predictions show good agreement with experimental results obtained for a particular configuration. Also, the results obtained with the proposed approach are compared favourably with general three-dimensional finite element computations, offering a reduction in the computational effort. Finally, a number of silencer geometries with granular material are considered. The effect of several parameters on the acoustic attenuation is assessed, including filling level, grain size and porosity.

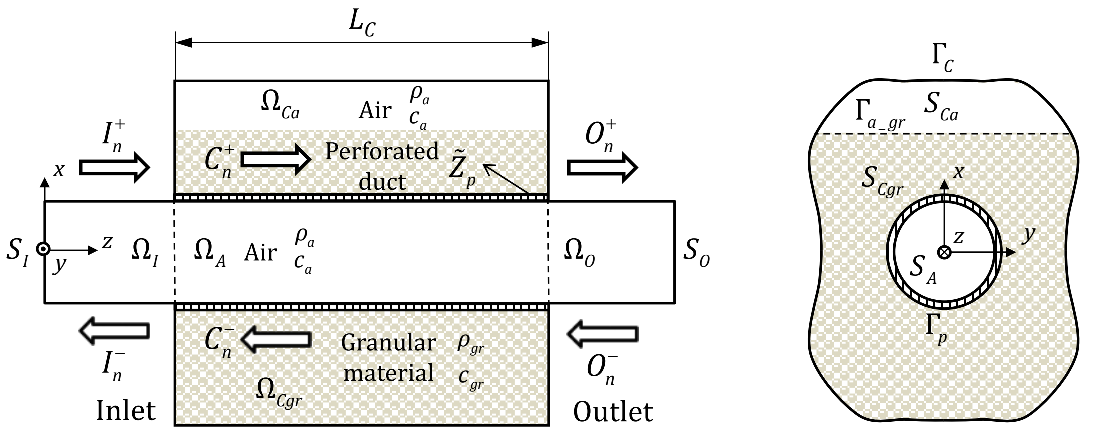

Figure 1. Scheme of silencer with granular material.

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3. References.

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