Calculation of the adjoint flux of the neutron diffusion equation

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The aim of this work is to perform a fast calculation of the adjoint flux of the neutron diffusion equation. The neutron diffusion equation is the easiest way of calculating the neutron distribution inside nuclear reactors. It is a partial differential equation, which contains spatial and time differential terms. Time differential terms are set to zero for considering the steady state of the neutron diffusion equation, which is an eigenvalue problem and the eigenvector is the neutron flux [1]. Spatial differential terms are discretized by using numerical methods in a discretized geometry.

The adjoint flux is the adjoint operator of the neutron diffusion equation and it can be understood as the neutron importance. The adjoint flux is important and useful in several applications, such as transient calculations and the generalized perturbation theory [1]. In these applications, both forward and adjoint neutron flux are needed, which can be calculated by solving both eigenvalue problems, forward and adjoint, of the steady state neutron diffusion equation.

These eigenvalue calculations might be computationally costly for large matrices. So, it would be desirable to calculate only one of them and calculate the other from the first one. It is known that the eigenvalues of the forward and adjoint problem are the same, but not the eigenvectors. In addition, forward and adjoint eigenvectors have a biorthogonal relationship for different eigenvalues.

There are some methods for estimating the adjoint eigenvectors from the forward ones, such as that developed in [2]. In this method, the adjoint eigenvectors are calculated by using a combination of the forward eigenvectors, whose coefficients are determined by solving a reduced eigenvalue problem. This method is fast, but it might be inaccurate for some cases.

In this work, the authors proposed a simpler and more accurate method, based on the product of the forward eigenvectors and the adjoint system matrix. In addition, the method includes a reorthogonalization for conserving the biorthogonal relationship of the forward and adjoint eigenvectors. Several tests are performed and compared with the method developed in [2] and with the adjoint eigenvalue calculation, which shows the capability of the method.

REFERENCES

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