Constructive analytic-numerical solution of random parabolic problems in a one-dimensional random medium by a mean square Fourier integral method

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This paper deals with the construction of analytic-numerical solution, in the mean square sense [1], of the time-dependent random parabolic partial differential problem

$$u_t(x,t) = a_2(t)\, u_{xx}(x,t) + a_1(t)\, u_x(x,t) + a_3(t)\, u(x,t)\,, \quad -\infty < x < +\infty,\ t > 0, \quad (1)$$

$$u(x,0) = f(x)\,, \quad -\infty < x < +\infty\,, \quad (2)$$

where $a_i(t) \equiv a_i(t,\omega) :]0,+\infty[\times\Omega \longrightarrow \mathbb{R}$, $1 \le i \le 3$ and $f(x) \equiv f(x,\omega) : \mathbb{R}\times\Omega \longrightarrow \mathbb{R}$ are stochastic processes (s.p.'s), defined in a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, that satisfy certain hypotheses.

For the random time-dependent coefficient parabolic models, the capture of the solution stochastic process (s.p.) of the original problem involves, throughout the inverse integral transform, unbounded random integrals that makes advisable the numerical evaluation of random complicated integrals. This is a major contribution introduced here, where we extend, to the random framework, the practical Gauss-Hermite quadrature formulae for the evaluation of improper random integrals that appear in a natural way when using random integral transform methods. We also show that, a random Fourier transform method can be applied so efficiently as it has been proved to be in the solution of deterministic problems [2]. However, in the random case, not only the solution s.p. is important, but also its expectation and standard deviation. To achieve these goals, firstly, we establish some new auxiliary results related to the so-called L^p-random calculus, [3], which are required throughout this work. Afterwards, problem (1)–(2) is solved analytically using the L^p-random calculus and the random Fourier transform approach introduced in [4]. Explicit expressions for the approximation of the expectation and the standard deviation of the solution s.p. of (1)–(2) are given. Finally, applying the random Gauss-Hermite quadrature formulae, numerical approximations of the expectation and the standard deviation are computed through an illustrative example.

References

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