Using Integer Linear Programming to study the relationship between the main variables involved in the construction of an external wall

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Abstract

This paper presents an Integer Linear Programming approach that takes into account some restrictions involving the construction of the external wall of a building: current legislation with respect to thermal transmittance, budget, thickness of the wall, number of layers, available materials and thicknesses for the different layers, workforce, time limits, etc. Among thousands of combinations of the different materials and thicknesses for the different layers that can make up the wall, the aim is to choose the best one to optimize an involved variable without violating any restriction to be taken into account in the construction of the wall. In particular, this paper deals with the problem of minimizing the thermal transmittance of the wall, in order to achieve a more sustainable and healthy indoor environment.

A case study consisting of a representative façade of 6 layers, with more than 670,000 possible combinations of materials and their thicknesses has been studied and evaluated in this paper. Seven scenarios of interest will be deeply discussed in the conclusions.

Keywords: Integer Linear Programming; thermal transmittance; external wall; building process; budget.

1. Introduction

Linear Programming, and particularly its cases with all integer variables (ILP) and with both integer and continuous variables, are increasingly applied in the field of energy and buildings to solve optimization problems, mainly to improve the energy efficiency of a building [1-4].

Thermal transmittance *U* ($Wm^{-2}K^{-1}$) ([5]) is a key magnitude to assess the energy efficiency of a building, and measures the heat that enters or leaves the building through the elements of the building envelope. The external wall is one of the most relevant parts of this envelope, and its thermal transmittance must abide by the current legislation [6] depending on the climate zone.

After an exhaustive research we realized that there is no work in the scientific literature relating the thermal transmittance of an external wall to be built, from the constructor's point of view, beyond the standards of the legal regulations. The aim of this paper is to present an ILP approach that takes into account some restrictions involving the building process of a wall: current legislation with respect to thermal transmittance, budget, thickness of the wall, number of layers, available materials and thicknesses for the different layers, workforce, time limits, final cost, etc. Among thousands of combinations of materials and thicknesses for the different layers of the wall, the aim of this paper is to choose the best one to optimize one of the involved variables without violating any restriction to be taken into account by the construction company. In particular, in this paper we deal with the problem of minimizing the thermal transmittance of the wall, but other variables, like cost or thickness, can be optimized in a similar way. This magnitude has been chosen due to the fact that the housing stock represents 24.8% of the final energy consumption in the EU [7] and due to an increased demand of a more sustainable and healthy indoor environment that considers the negative effects related with climate changes.

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Nomenclature

n	Number of layers of the wall
s	Total surface in m^2 of the wall
m	Number of different materials
h_{int}	Standard internal conductivity
h_{ext}	Standard external conductivity
w_j	Number of different thicknesses for material *j*
e_{jk}	Thickness corresponding to material *j* with type of thickness *k*
c_{ijk}	Cost of placing in layer *i* $1m^2$ of material *j* with type of thickness *k*
t_{ijk}	Time of placing in layer *i* $1m^2$ of material *j* with type of thickness *k*
e_{min}	Lower bound for the thickness of the wall
e_{max}	Upper bound for the thickness of the wall
s_{max}^{jk}	Maximum number of m^2 available of material *j* with thickness of type *k*
b_{max}^{jk}	Maximum budget for the installation of the material *j* with thickness of type *k*
t_{max}	Maximum time required to construct the wall
b_{max}	Maximum budget to construct the wall
U	Thermal transmittance
U_{max}	Maximum thermal transmittance allowed for the wall
λ_j	Thermal conductivity corresponding to material *j*

2. Definition of the ILP problem

Taking into account the nomenclature given above, the variables of the ILP problem are x_{ijk}, whose values are 1 if layer *i* is made with material *j* and thickness *k*, or 0 otherwise, $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m\}$, $k \in \{1, \dots, w_j\}$. It is important to stress that *k* does not indicate the measure of the thickness but the type of thickness. Note also that layers will be enumerated from outside to inside.

The thermal transmittance of the wall, as described in [5], is therefore given by Eq. (1):

$$U = \frac{1}{\frac{1}{h_{int}} + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{w_j} \frac{e_{jk}}{\lambda_j} x_{ijk} + \frac{1}{h_{ext}}} \qquad (1)$$

Since *U* is not a linear function of variables x_{ijk}, it cannot be the objective function of the ILP problem. However, h_{int}, h_{ext}, e_{jk} and λ_j are constant for all the involved subscripts, and minimizing *U* is equivalent to maximizing the triple summation given in the denominator of *U*, which is certainly a linear function of binary variables x_{ijk}. Therefore, our ILP problem will maximize this triple summation. The restriction set of the ILP problem is open, in the sense that the set of restrictions presented represents the most usual conditions imposed to a constructor to build an external wall. But this set can be expanded, reduced or modified, according to the particular conditions or interest of each building in construction or refurbishment, to adjust as much as possible the mathematical model to the real problem.

The ILP formulation of the problem studied here is given through Eqs. 2 to 12:

$$Maximize \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{w_j} \frac{e_{jk}}{\lambda_j} x_{ijk} \qquad (2)$$

s.t.:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{w_j} \frac{e_{jk}}{\lambda_j} x_{ijk} \geq \frac{1}{U_{max}} - \frac{1}{h_{int}} - \frac{1}{h_{ext}} \qquad (3)$$

$$e_{min} \leq \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{w_j} e_{jk} x_{ijk} \leq e_{max} \qquad (4)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{w_j} c_{ijk} x_{ijk} \leq b_{max} \qquad (5)$$

$$\sum_{j=1}^{m}\sum_{k=1}^{w_j} x_{ijk} = 1 \quad \forall i \in \{1, \dots, n\} \tag{6}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{w_j} st_{ijk}x_{ijk} \le t_{max} \tag{7}$$

$$\sum_{i=1}^{n} sx_{ijk} \le s_{max}^{jk} \quad \forall j \in \{1, \dots, m\}, k \in \{1, \dots, w_j\} \tag{8}$$

$$\sum_{i=1}^{n} sc_{ijk}x_{ijk} \le b_{max}^{jk} \quad \forall j \in \{1, \dots, m\}, k \in \{1, \dots, w_j\} \tag{9}$$

$$x_{ijk} = 0 \quad \forall\, ijk - incompatible \tag{10}$$

$$x_{ijk} + x_{(i+1)j'k'} \le 1 \quad \forall\, (ijk - (i+1)j'k') - \ incompatible \tag{11}$$

$$x_{ijk} \in \{0,1\} \quad \forall\, i \in \{1, \dots, n\}, j \in \{1, \dots, m\}, k \in \{1, \dots, w_j\} \tag{12}$$

In this formulation:

- Eq. (3) ensures that the obtained thermal transmittance meets the legal upper bound U_{max} according to the climate zone.
- Eq. (4) guarantees that the total thickness of the wall belongs to the interval $[e_{min}, e_{max}]$.
- Eq. (5) forbids that the cost per m^2 of the wall exceeds the budgeted cost b_{max}.
- Eq. (6) guarantees that each layer is composed by exactly one material with a specific thickness.
- Eq. (7) forbids to exceed the established time limit t_{max} to build a m^2 of the wall.
- Eq. (8) takes into account the available quantity of each material with its respective thicknesses.
- Eq. (9) forbids to spend more money than budgeted for each material and thickness.
- Eq. (10) forbids to place a given material *j* with a given thickness *k* in a given layer *i* (this fact is denoted by "*ijk*-incompatibility"). For instance, it does not make sense to put a waterproof extruded face brick in an intermediate layer. But even if some options make sense, the conditions imposed on the constructor may forbid these options.
- Eq (11) forbids to place a material *j'* with thickness *k'* in the next layer to the one (layer *i*) containing the material *j* with thickness *k* (this fact is denoted by (*ijk*-(*i*+1)*j'k'*)-incompatibility). Therefore, at most one of the two materials with the given thickness will appear in the corresponding layer. For instance, it does not make sense to put solid concrete block as a layer, with the next layer (to the interior) made by pressed face brick.
- Eq. (12) defines variables x_{ijk} as binary.

3. Case study and results

We present a case study consisting of a façade of 6 layers. This façade is a common but representative constructive solution. We describe next the composition of the external wall (layers, materials, thicknesses and fixing solutions). Layer 1 (external coating): 2 plaster types, plates, absence. Layer 2 (external panel): solid brick, concrete block, face brick, 2 pressed face brick. Layer 3 (air chamber): light ventilated, not ventilated, absence. Layer 4 (thermal insulation): 6 materials, 4 thicknesses, 3 fixing methods. Layer 5 (internal panel): solid, air, or perforated brick. Layer 6 (internal coating): plaster with 4 thicknesses. In all cases costs are taken from the cost generator website of CYPE Ingenieros [8]. Costs always include materials, staff and site facilities. With these options, a total amount of 671,328 combinations for this external wall are possible.

Furthermore, the recommended thermal resistance for the air layers close to the external and internal surfaces are: $1/h_{ext}$ = 0,04 m^2KW^{-1} and $1/h_{int}$ = 0,13 m^2KW^{-1} as indicated in the Spanish Technical Act (CTE), Basic Document of Energy Saving (DB_HE) [6].

We have considered that the total thickness of the wall can vary between 0.24 and 0.69 m in intervals of 1 cm. We have also considered a budget to construct 1 m^2 of wall limited to an amount ranging between €85 and €190, with intervals of €5. The aim is to find the lowest thermal transmittance wall for each combination of wall thickness and budget. As there are 45 intervals of 1 cm and 22 budgets, 990 ILP problems have been solved using *Mathematica* 10.4 [9].

As expected trend, given a fixed thickness, the thermal transmittance decreases as the budget increases, although the variability of *U* is usually only a few decimals. Another expected trend is that given a budget, the thermal transmittance also decreases as the thickness increases, but once a certain thickness is exceeded, the problem becomes impossible.

Obviously, for each optimal solution, in addition to its thermal transmittance, *Mathematica* provides the values of the binary variables, so it is easy to see which is the chosen material and its thickness for each one of the six layers in the solution with minimum thermal transmittance. Through Table 1 we show the main data corresponding to the best solution obtained under 7 different scenarios, which represent the extreme cases: minimum budget (90 €/m^2); minimum thickness (interval [0.24,0.25m]); maximum budget (up to 190 €/m^2); maximum thickness (interval [0.68,0.69m]); given the minimum budget, take the minimum possible thickness; given the minimum thickness, take the minimum possible budget; and finally, the combination budget/thickness that has obtained the optimal solution with lowest thermal transmittance.

Table 1
Optimal solution for 7 scenarios of interest.

	Minimum Budget	Minimum thickness	Maximum budget	Maximum thickness	Minimum budget / minimum thickness	Minimum thickness / minimum budget	Minimum *U*
Budget	90	130	190	190	90	125	190
Thickness interval	[0.46,0.47]	[0.24,0.25]	[0.66,0.67]	[0.68,0.69]	[0.27,0.28]	[0.24,0.25]	[0.66,0.67]
Minimum *U*	0.243525	1.33167	0.203537	0.204624	0.706436	1.345458	0.203537
Exact cost	89.71	125.22	166.89	189.18	89.57	123.82	166.89
Exact thickness	0.463	0.25	0.664	0.682	0.278	0.248	0.664

As the most relevant data shown by Table 1, with our selection of materials and thicknesses, the lowest possible *U* value for an external wall is 0.2035 $Wm^{-2}K^{-1}$, which is achieved for a cost of 166.89 €/m^2 and a thickness of 0.664m. Note that this *U* value is very small and therefore is useful for every climate zone, but we can see that an *U* value of 0.2435 $Wm^{-2}K^{-1}$ (only four hundredths more, but also useful for every climate zone), can be achieved with cost 89.71 €/m^2 and a thickness of 0.466m. We want to point out finally a very important result of this study: the building company can considerably reduce the cost of the external wall with a very low increase of the thermal transmittance, just choosing the right materials and thicknesses in the correct constructive configuration.

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