Risk-neutral processes estimation in interest rate models with stochastic volatility and jumps

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Traditionally, the financial literature assumes that interest rates move continuously and they are modelled as diffusion processes, as in Vasiceck (1977), Cox *et al* (1985) and so on. However, more recent studies have showed that interest rates contained unexpected discontinuous changes, see for example Das (2002) and Johannes (2004). Jumps in interest rates, probably, are due to different market phenomena such as surprises or shocks in foreign exchange markets. Moreover, when pricing and hedging financial derivatives jump diffusion models are very important, since ignoring jumps can produce hedging and pricing risks, see Lin and Yeh (1999).

It is widely known that one-factor interest rate models are very attractive for practitioners because its simplicity and computational convenience. However, these models have also unrealistic properties. First, they cannot generate all the yield curve shapes and changes that we can find in the markets. Second, the changes over infinitesimal periods of any two interest-rate dependent variables will be perfectly correlated. Finally, as Hong and Li (2005) show, none of their analyzed one-factor models adequately captures the interest rate dynamics. Therefore, we consider that at least two factors are necessary to model the term structure of interest rates. In fact, the number of factor must be a compromise between numerical efficient implementation and the capability of the model to fit data.

In this paper, we consider a two-factor interest rate model: the instantaneous interest rate (r) and the volatility (v). We assume that under the risk-neutral measure the interest rate follows a jump-diffusion process and the volatility a diffusion process as follows,

$$
\begin{aligned}
dr &= \left(\mu_r - v\theta^{W_r^{\mathcal{Q}}} + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[J]\right) dt + \sqrt{v} dW_r^{\mathcal{Q}} + J d\tilde{N}^{\mathcal{Q}}, \\
dv &= \left(\mu_v - \sigma_v \theta^{W_v^{\mathcal{Q}}}\right) dt + \sigma_v dW_v^{\mathcal{Q}},
\end{aligned}
$$

where μ_r and μ_v are the drifts and σ_v the volatility of interest rate volatility. The jump amplitude J is a function of r, v and Y, where Y is a random variable with probability distribution Π. Moreover, $W_r^{\mathcal{Q}}$ and $W_v^{\mathcal{Q}}$ are the Wiener processes under $\mathcal{Q}$-measure and $N^{\mathcal{Q}}$ represents a Poisson process, under $\mathcal{Q}$ measure with intensity $\lambda^{\mathcal{Q}}(r, v)$. We assume that $W_r^{\mathcal{Q}}$ and $W_v^{\mathcal{Q}}$ are independent of $N^{\mathcal{Q}}$ but the standard Brownian motions are correlated with:

$$
Cov(W_X^{\mathcal{Q}}, W_\delta^{\mathcal{Q}}) = \rho t.
$$

The market prices of risk associated to $W_r^{\mathcal{Q}}$ and $W_v^{\mathcal{Q}}$ Wiener processes are $\theta^{W_r^{\mathcal{Q}}}(r, v)$ and $\theta^{W_v^{\mathcal{Q}}}(r, v)$, respectively.

In the literature, the functions of the stochastic processes and the market prices of risk are usually specified as simple parametric functions for pure simplicity and tractability for obtaining a closed-form solution for the zero-coupon bond prices. Then, several methods such as GMM or Kalman Filter can be used to estimate the whole functions, see for example Jiang and Yan (2009). However, there is neither evidence nor consensus that these affine or linear-quadratic models where accurate to explain the yield curves. Therefore, in this paper we propose a nonparametric jump-diffusion model which avoids imposing arbitrary restrictions to the functions.

In the spirit of studies such as Gómez-Valle and Martínez-Rodríguez (2015) and Gómez-Valle and Martínez-Rodríguez (2016) in one-factor interest rate models and Gómez-Valle et al. (2017) in commodity derivatives pricing who provide some relations to estimate the risk-neutral functions directly from data in the markets, we propose a novel approach to estimate the whole functions of a two factor jump-diffusion term structure model directly from market data, although a closed-form solution for the zero-coupon bond prices is not known. As we estimate directly the risk-neutral functions from interest rate data, the market prices of risk has not to be arbitrary specified. Moreover, this approach can be applied to parametric as well as nonparametric models.

In order to implement our approach, we take interest rate data from the Federal Reserve in USA. Our empirical results show that a two-factor model estimated with our approach is more accurate than the jump-difussion one-factor model. Moreover, the higher the maturity the higher the differences.

Finally, we also find that meanwhile the one-factor model, in general, underprices the yield curves, our two-factor model slightly overprices. This fact should be taken into account by practitioners in the markets for pricing and hedging interest rate derivatives.

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References

J. Cox, J. Ingersoll and S. Ross, An intertemporal general equilibrium model of asset prices, Econometrica 53 (1985) 363-384.

S. R. Das, The surprise element: Jumps in interest rates, J. Econom. 79 (2002) 507-576.

L. Gómez-Valle, J. Martínez-Rodríguez, Advances in pricing commodity futures: Multifactor models, Math. Comp. Model. 57 (2013) 1722-1731.

L. Gómez-Valle, J. Martínez-Rodríguez, The Role of the Risk-Neutral Jump Size Distribution in Single-Factor Interest Rate Models, Abstr. Appl. Anal. Volume 2015, Article ID 805695 (2015) 1-8.

L. Gómez-Valle, J. Martínez-Rodríguez, Estimation of risk-neutral processes in single-factor jump-diffusion interest rate models, J. Comput. Appl. Math. 291 (2016) 48-57.

L. Gómez-Valle, Z. Habibilashkary, J. Martínez-Rodríguez, A new technique to estimate the risk-neutral processes jump-diffusion commodity futures models, J. Comput. Appl. Math. 309 (2017) 435-441.

B. H. Lin and S. K. Yeh, Jump diffusion interest rate process: an empirical examination, J. Bus. Financ. Acc. 26 (1999) 967-995.

Y. Hong and H. Li, Nonparametric specification testing for continuous-time models with applications to term structure ofinterest rates, R. Financ. Stud. 18 (2005) 37-84.

G. Jiang and S. Yan, Linear-quadratic term structure models-Toward the understanding of jumps in interest rates, J. Bank. Financ. 33 (2009) 473-485.

M. Johannes, The statistical and economic role of jumps in continuous-time interest rate models, J. Finance 59 (2004) 227-259.

O. Vasiceck, An equilibrium characterization of the term structure, J. Financ. Econom. 5 (1977) 177-188.