# Efficient class of iterative schemes with memory for solving nonlinear problems *

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Iterative methods are widely used for solving nonlinear problems that arise from the modelization of real processes. These nonlinear problems can be one or multidimensional, but they are described by a nonlinear function $F : \mathbb{R}^n \to \mathbb{R}^n$, being $n \geq 1$. The most used method for approximating the roots of $F(x) = 0$ is Newton's scheme, whose iterative expression is

$$x^{(k+1)} = x^{(k)} - [F'(x^{(k)})]^{-1}F(x^{(k)}),\ k \geq 0,$$

where $F'(x^{(k)})$ denotes the Jacobian matrix associated to the nonlinear function $F$ at iterate $x^{(k)}$. This general use is due to its simplicity and economy of operations and functional evaluations. However, in last years some multi-step methods have been designed increasing the efficiency of Newton's procedure in these aspects, see for example [3–5, 9] and the references therein. In [1], the authors designed the most efficient method for solving nonlinear systems, as per our knowledge, in terms of computational efficiency, as it reaches fifth-order of convergence with one evaluation of the associated Jacobian matrix, three functional evaluations and only one inverse operator. This scheme can also be found as a particular element of a parametric family, denoted by M4, whose stability is analyzed in [2]. This class is the starting point of the present work.

$$\begin{aligned}
y^{(k)} &= x^{(k)} - [F'(x^{(k)})]^{-1}F(x^{(k)}), \\
z^{(k)} &= y^{(k)} - \frac{1}{\beta}[F'(x^{(k)})]^{-1}F(y^{(k)}), \\
x^{(k+1)} &= z^{(k)} - [F'(x^{(k)})]^{-1}\left(\left(2 - \frac{1}{\beta} - \beta\right)F(y^{(k)}) + \beta F(z^{(k)})\right).
\end{aligned}$$

In recent years, several multi-step iterative methods with memory have been developed. This kind of iterative methods uses information from the current and previous iterations and can increase

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the convergence order and the computational efficiency of the multi-step iterative methods without memory with no additional functional evaluations. The increasing in the order of convergence is based on one or more accelerating parameters which appear in a factor of the error equation of the method without memory. For a background study regarding the acceleration of convergence order via with memorization, one should see e.g. [6, 7].

Only very recently, some iterative schemes with memory for solving nonlinear systems have been designed (see [8]). Our main aim is to introduce some accelerating parameters in the iterative expressions such that, holding the order of convergence of M4, the final error equation allows us to accelerate the order of convergence by using functional evaluations of previous iterates. The obtained results show that, not only the order of convergence is increased but also its efficiency is improved, as no new functional evaluation is added. Moreover, the "memorizing" process is made without eliminating the Jacobian matrices in the iterative expressions. This fact yields in a better stability and greater sets of feasible starting points.

# References


[1] V. Arroyo , A. Cordero , J.R. Torregrosa , Approximation of artificial satellites preliminary orbits: the efficiency challenge, Math. Comput. Model. 54 (2011) 18021807 .

[2] A. Cordero, J.M. Gutiérrez, Á.A. Magreñán, J.R. Torregrosa, Stability analysis of a parametric family of iterative methods for solving nonlinear models, Appl. Math. Comput. 285 (2016) 26-40.

[3] A. Cordero, J.G. Maimó, J.R. Torregrosa, M.P. Vassileva, Solving nonlinear problems by OstrowskiChun type parametric families, Math. Chem. 53 (2015) 430–449.

[4] A. Cordero, J.R. Torregrosa, M.P. Vassileva, Design, Analysis and Applications of Iterative Methods for Solving Nonlinear Systems. Nonlinear Systems- Design, Analysis, Estimation and Control, 87-116, Intech 2016.

[5] J.L. Hueso, E. Martínez, C. Teruel, Convergence, effiency and dinamimics of new fourth and sixth order families of iterative methods for nonlinear systems, Comput. Appl. Math. 275 (2015) 412–420.

[6] M.S. Petković, B. Neta, L.D. Petković, J. Džunić, Multipoint Methods for Solving Nonlinear Equations, Academic Press, Amsterdam, 2013.

[7] M.S. Petković, B. Neta, L. D. Petković, J. Džunić, Multipoint methods for solving nonlinear equations: a survey, Appl. Math. Comput. 226 (2014) 635–660.

[8] M.S. Petković, J.R. Sharma, On some efficient derivative-free iterative methods with memory for solving systems of nonlinear equations, Numer. Algor. 71(2) (2016) 457–474.

[9] J.R. Sharma, H. Arora, Efficient Jarratt-like methods for solving systems of nonlinear equations, Calcolo 51 (2014) 193–210.