Modal Method for the Efficient Analysis and Design of Microwave Filters based on Multiple Discontinuities

Juan R. Sánchez, Carmen Bachiller, María Juliá and Héctor Esteban
Instituto de Telecomunicaciones y Aplicaciones Multimedia.
Universitat Politècnica de València.
Camino de Vera, s/n, 46021 Valencia, Spain.
e-mail: juasncm1@upv.es

I. Introduction

Microwave filters are essential components in high frequency communication systems, whose required features are increasing in the recent years. Among these features, the need for low cost devices with a reduced mass and volume and the need to integrate them with the current planar technology are highlighted.

So far, for designing these filtering structures, different technologies are used. One of them is based on rectangular o circular metallic waveguides, which present low insertion losses, ability to carry out high power signals and high quality factor [1]. However, metallic waveguides are heavy, big, expensive and difficult to integrate with planar technology. Lately Substrate Integrated Waveguide (SIW) technology has appeared to solve these problems [2]. It integrates a rectangular waveguide into a planar substrate, obtaining devices much smaller, significantly cheaper and easier to manufacture.

No matter which waveguide technology is applied, the coupled cavities H-plane filters are the most used. Their design combines cavity resonators and impedance inverters. These impedance inverters are implemented by coupling windows, whose widths change depending on the desired impedance value [3]. However, there is a new alternative to this scheme consisting on filling the waveguide with dielectric material by sections, keeping the width of the whole structure.

There are several commercial tools like Ansys HFSS [4] and CST Studio Suite [5] based on numerical methods that enable to carry out the analysis and design of these structures, but they require a very high computational time during the analysis process. This affects negatively to the automated design of these devices, since the optimizatin process of the design requires a huge number of iterations of the analysis of the structure.

In this paper, the authors propose an efficient and accurate analysis method by following a multimodal analysis of the device. The method allows calculating the matrix with the scattering parameters of the filter for M modes. This multimodal scattering matrix gives, for each mode - or solution of the wave equation of the electromagnetic field -, the transmission and reflection parameters of the structure, i.e. its electromagnetic behavior. This method is far more accurate and efficient than the numerical methods used by the commercial tools.

II. Analysis of the filter

The device is considered as a waveguide with N different sections of length $l^{(i)}$ and $N-1$ dielectric discontinuities as shown in Fig. 1. The sections filled with a dielectric material (odd sections in the figure, except the first and the last one) behave as resonant cavities and the empty sections (even ones in the figure) are the coupling windows.

For the analysis, the input and output normalized electric and magnetic fields are defined for each section and mode. At each discontinuity, the continuity of electric and magnetic transverse fields is forced for the M modes, obtaining $2(N-1)$ equations for each mode. These equations are solved recursively obtaining the cross relation between the input and output voltages of the first and last section. The relations of these normalized voltages determine the scattering matrix (**S**) of the whole device.

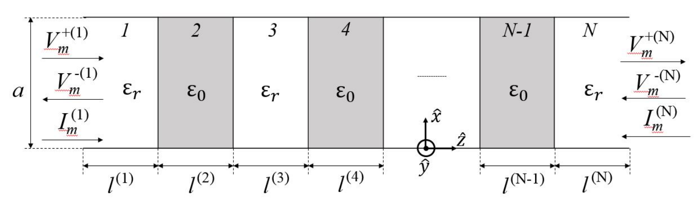

Fig. 1. Multiple discontinuities waveguide filter and its reference system. Gray sections are the coupling windows. White sections are the resonant cavities.

Since both the geometry and the excitation are invariant in height (dimension y), for the analysis, only TE_{m0} modes are considered. Furthermore, although there are infinite modes, only the first M modes are selected.

The electric ($\overrightarrow{E}_t^{(i)}$) and magnetic ($\overrightarrow{H}_t^{(i)}$) transverse fields of each section i of the waveguide are defined by (1) and (2), respectively [6].

$$\overrightarrow{E}_t^{(i)} = \sum_{m=1}^{M} V_m^{(i)}(z) \overrightarrow{e}_m^{(i)}(x,y) \tag{1}$$

$$\overrightarrow{H}_t^{(i)} = \sum_{m=1}^{M} I_m^{(i)}(z) \overrightarrow{h}_m^{(i)}(x,y) \tag{2}$$

On the one hand, $V_m^{(i)}(z)$ and $I_m^{(i)}(z)$ are, respectively, the amplitude and the current of the wave in each section composed by the incident and reflected field. On the other hand, $\overrightarrow{e}_m^{(i)}$ and $\overrightarrow{h}_m^{(i)}$ are the m-th modes in the i-th section.

$$\begin{gathered} V_m^{(i)}(z_i) = V_m^{+(i)} e^{-\gamma_m^{(i)} z_i} + V_m^{-(i)} e^{+\gamma_m^{(i)} z_i} \\ \overrightarrow{e}_m^{(i)} = -\hat{y}\sqrt{\tfrac{2Z_{0m}^{(i)}}{a}} sin\left(\tfrac{m\pi x}{a}\right) \end{gathered} \tag{3}$$

$$\begin{gathered} I_m^{(i)}(z_i) = \tfrac{V_m^{+(i)}}{Z_{0m}^{(i)}} e^{-\gamma_m^{(i)} z_i} - \tfrac{V_m^{-(i)}}{Z_{0m}^{(i)}} e^{+\gamma_m^{(i)} z_i} \\ \overrightarrow{h}_m^{(i)} = \hat{x}\sqrt{\tfrac{2Z_{0m}^{(i)}}{a}} sin\left(\tfrac{m\pi x}{a}\right) \end{gathered} \tag{4}$$

In these equations

- m and i are, respectively, the index of the correspondent guided mode and the index of the section.
- M is the number of guided modes.
- a is the width of the structure. In the structure considered in this work a is the same for all the sections i.

For an even section, the dielectric permittivity of the medium is that of the vacuum, but in odd sections, there is a different dielectric, so that:

$$Z_{0m}^{(i)} = \begin{cases} \frac{j\omega\mu}{\gamma_{mE}^{(i)}} & \textit{For even i} \\ \frac{j\omega\mu}{\gamma_{mO}^{(i)}} & \textit{For odd i} \end{cases} \tag{5}$$

$$\gamma_m^{(i)} = \begin{cases} \frac{2\pi}{\lambda_0}\sqrt{\left(\frac{m}{2a/\lambda_0}\right)^2 - 1} & \textit{For even i} \\ \frac{2\pi}{\lambda_0}\sqrt{\left(\frac{m}{2a/\lambda_0}\right)^2 - \varepsilon_r} & \textit{For odd i} \end{cases} \tag{6}$$

After the analysis of these equations, the next problem is obtained:

- $2N$ unkwons, since there are $V_m^{+(i)}$ and $V_m^{-(i)}$ for $i = 1, 2, \ldots N$
- $2(N-1)$ equations, since transverse fields continuity is forced, $J_{c,i} = 0$, in $N-1$ discontinuities

There are $2(N-1) = 2N-2$ equations for $2N$ unkwons, so $V_m^{+(1)}$ and $V_m^{-(N)}$ have to be figured out. They are given by:

$$V_m^{(1)} = Z_m^{(1,1)} I_m^{(1)} + Z_m^{(1,N)} I_m^{(N)} \tag{7}$$

$$V_m^{(N)} = Z_m^{(N,1)} I_m^{(1)} + Z_m^{(N,N)} I_m^{(N)} \tag{8}$$

where $V_m^{(1)}$ and $I_m^{(1)}$ depends on $V_m^{+(1)}$ and $V_m^{-(1)}$, and $V_m^{(N)}$ and $I_m^{(N)}$ depends on $V_m^{+(N)}$ and $V_m^{-(N)}$ as seen before.

To obtain the $\mathbf{S}$ matrix, $V_m^{+(N)}$ and $V_m^{-(N)}$ have to be related with $V_m^{+(1)}$ and $V_m^{-(1)}$ by forcing continuity of electric and magnetic transvere fields on each discontinuity.

As $\overrightarrow{e}_m(x,y)$ and $\overrightarrow{e}_n(x,y)$ as well as $\overrightarrow{h}_m(x,y)$ and $\overrightarrow{h}_n(x,y)$ with $m \neq n$ are orthogonal, if the equations of electric and magnetic fields are scalarly multiplied by $\overrightarrow{e}_n(x,y)$ and $\overrightarrow{h}_n(x,y)$ respectively, it is obtained:

$$V_m^{+(i)} e^{-\gamma_m^{(i)} l^{(i)}} + V_m^{-(i)} e^{+\gamma_m^{(i)} l^{(i)}} = V_m^{+(i+1)} + V_m^{-(i+1)} \tag{9}$$

$$\frac{V_m^{+(i)}}{Z_{0m}^{(i)}} e^{-\gamma_m^{(i)} l^{(i)}} - \frac{V_m^{-(i)}}{Z_{0m}^{(i)}} e^{+\gamma_m^{(i)} l^{(i)}} = \frac{V_m^{+(i+1)}}{Z_{0m}^{(i+1)}} - \frac{V_m^{-(i+1)}}{Z_{0m}^{(i+1)}} \tag{10}$$

Doing (9) + (10)$\cdot Z_{0m}^{(i+1)}$ and (9) − (10)$\cdot Z_{0m}^{(i+1)}$:

$$\begin{pmatrix} V_m^{+(i+1)} \\ V_m^{-(i+1)} \end{pmatrix} = \mathbf{A^{(i)}} \begin{pmatrix} V_m^{+(i)} \\ V_m^{-(i)} \end{pmatrix} \tag{11}$$

where

$$\mathbf{A^{(i)}} = \frac{1}{2}\left(\begin{array}{c|c} e^{-\gamma_m^{(i)} l^{(i)}}\left(1 + \frac{Z_{0m}^{(i+1)}}{Z_{0m}^{(i)}}\right) & e^{+\gamma_m^{(i)} l^{(i)}}\left(1 - \frac{Z_{0m}^{(i+1)}}{Z_{0m}^{(i)}}\right) \\ \hline e^{-\gamma_m^{(i)} l^{(i)}}\left(1 - \frac{Z_{0m}^{(i+1)}}{Z_{0m}^{(i)}}\right) & e^{+\gamma_m^{(i)} l^{(i)}}\left(1 + \frac{Z_{0m}^{(i+1)}}{Z_{0m}^{(i)}}\right) \end{array}\right)$$

The number of discontinuities is a multiple of 2 and N is odd, since the first and the last mediums are different from vacuum. The relation between the input and output waves in the first and last section is:

$$\begin{pmatrix} V_m^{+(N)} \\ V_m^{-(N)} \end{pmatrix} = \underbrace{\prod_{k=\frac{N-1}{2}}^{1} \mathbf{A^{(2k)}} \mathbf{A^{(2k-1)}}}_{\mathbf{A}} \begin{pmatrix} V_m^{+(1)} \\ V_m^{-(1)} \end{pmatrix} \tag{12}$$

By definition and considering the diagram in Fig. 1, the values of the $\mathbf{S}$ parameters depending on the input and output waves can be expressed as:

$$\begin{aligned} V_m^{-(1)} &= S_{11} V_m^{+(1)} + S_{12} V_m^{-(N)} \\ V_m^{+(N)} &= S_{21} V_m^{+(1)} + S_{22} V_m^{-(N)} \end{aligned} \tag{13}$$

By operating the expressions (12) in order to obtain similar equations to (13), it is obtained that:

$$V_m^{-(1)} = \underbrace{-A_{22}^{-1} A_{21}}_{S_{11}} V_m^{+(1)} + \underbrace{A_{22}^{-1}}_{S_{12}} V_m^{-(N)} \tag{14}$$

$$V_m^{+(N)} = \underbrace{A_{11} - A_{12} A_{22}^{-1} A_{21}}_{S_{21}} V_m^{+(1)} + \underbrace{A_{12} A_{22}^{-1}}_{S_{22}} V_m^{-(N)} \tag{15}$$

By comparing (14) and (15) with the equations (13) the values of the matrix $\mathbf{S_m}$ may be determined, as indicated in the above expressions. Thus, each mode of the $\mathbf{S}$ matrix is expressed as follows:

$$\mathbf{S_m} = \left(\begin{array}{c|c} -A_{22}^{-1} A_{21} & A_{22}^{-1} \\ \hline A_{11} - A_{12} A_{22}^{-1} A_{21} & A_{12} A_{22}^{-1} \end{array}\right) \tag{16}$$

The multimodal $\mathbf{S}$ matrix can be written as:

$$
\mathbf{S} = \left(\begin{array}{cccc|cccc}
S_1^{(1,1)} & 0 & \cdots & 0 & S_1^{(1,N)} & 0 & \cdots & 0 \\
0 & S_2^{(1,1)} & \cdots & 0 & 0 & S_2^{(1,N)} & \cdots & 0 \\
\vdots & \ddots & & \vdots & | & \ddots & & \vdots \\
0 & \cdots & S_{M-1}^{(1,1)} & 0 & 0 & \cdots & S_{M-1}^{(1,N)} & 0 \\
0 & \cdots & 0 & S_M^{(1,1)} & 0 & \cdots & 0 & S_M^{(1,N)} \\
\hline
S_1^{(N,1)} & 0 & \cdots & 0 & S_1^{(N,N)} & 0 & \cdots & 0 \\
0 & S_2^{(N,1)} & \cdots & 0 & 0 & S_2^{(N,N)} & \cdots & 0 \\
\vdots & \ddots & & \vdots & \vdots & \ddots & & \vdots \\
0 & \cdots & S_{M-1}^{(N,1)} & 0 & 0 & \cdots & S_{M-1}^{(N,N)} & 0 \\
0 & \cdots & 0 & S_M^{(N,1)} & 0 & \cdots & 0 & S_M^{(N,N)}
\end{array} \right) \tag{17}
$$

This matrix is $2M \times 2M$, since each block is composed by one $M \times M$ matrix. Moreover, they are diagonal matrices, so $\mathbf{S}$ is also diagonal in blocks.

III. Results

The analysis tool is implemented in MatLab and integrated into a Computer Aided Design (CAD) tool based on the theoretical synthesis of a starting point, in order to calculate the physical parameters of the stucture, i.e. the lengths of the resonant cavities and the coupling windows. Once the starting point has been calculated, the next step is the optimization of these design dimensions to get a frequency response as much similar as possible to the ideal response.

The optimization process is based on several iterations of the analysis of the structure: in each iteration the analysis tool calculates the electromagnetic response of the structure, then it is compared to the desired response, if the difference between them does not reach a predefined minimun, the structure parameters are slightly changed and the process is reapeated. Obviously, the strategy for changing these physical dimmensions is not random, but a set of optimizations algorithms are used. The whole process is described in [7], where its efficiency and robustness are improved by using the adequate combination of algorithms, i.e. the hybridization of Direct Search with Coordinate Rotation [8], Downhill Simplex Method [9] and Broyden Fletcher Goldfarb Shanno (BFGS) [10] methods.

In order to evaluate the effectiveness (computational time) and the accuracy (frequency response) of the analysis tool, several filters with different number of cavities have been designed and afterward analyze. The responses of two of them are presented.

Fig. 2 shows the frequency response of a filter of two cavities when it is analyzed with the analysis tool and a commercial software (CST). Both modal and numerical analysis are compared to the ideal response of a two cavities Chebysev filter. It is observed that there is a very good agreement between all of them. It means that the developed analysis tool is accurate.

Fig. 3 shows the same comparison that the previous figure but for a filter of four cavities. In this case, it is also observed the very good agreement between all the frequency responses. The analysis tool can be used for filters of different orders.

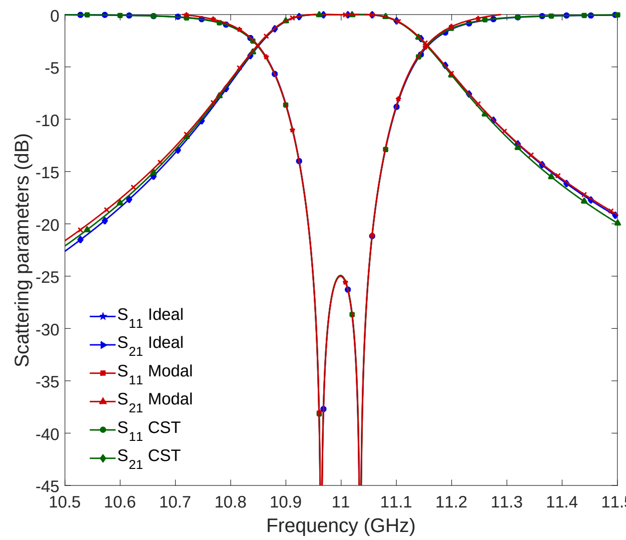

Fig. 2. Comparison of frequency responses for a filter of two cavities.

Once the accuracy of the tool is checked, the effectiveness is also tested. The computational time for the analysis of filters from two to ten cavities is shown in Table I for the Modal Method and CST commercial software. This time is calculated under the same conditions for both: computing processor, start and stop frequencies and number of sample points. The obtained times are plotted in Fig. 4. It is observed that the computational time of the commercial software based on numerical methods increases linearly with a sharp slope when the number of cavities increases. However, in the case of the developed modal method the time increases very slowly. It shows the advantage of the new method versus the commercial one.

TABLE I
Computational time for the analysis of the filter.

Cavities	Modal Method	CST
2	2.70 s	40.00 s
4	4.28 s	80.00 s
6	5.79 s	130.00 s
8	7.15 s	160.00 s
10	8.85 s	194.00 s

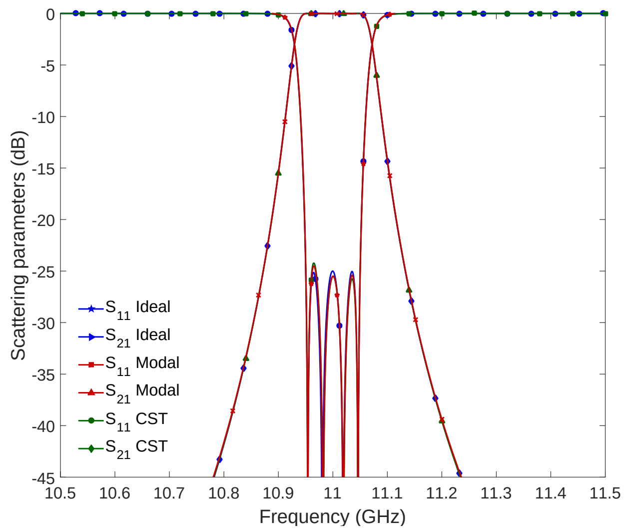

Fig. 3. Comparison of frequency responses for a filter of four cavities.

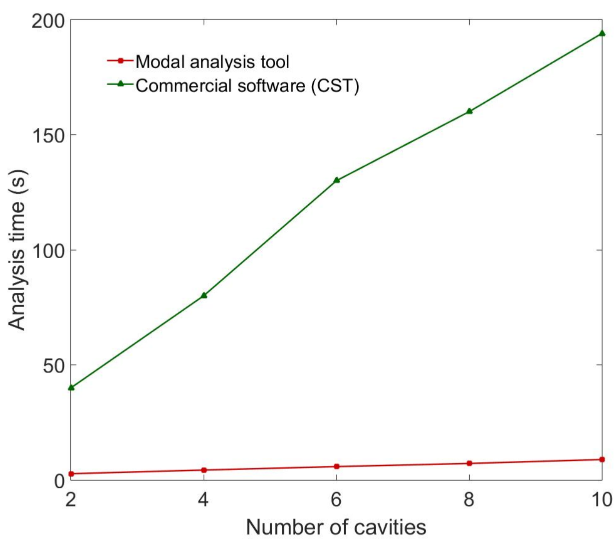

Fig. 4. Computational time comparison between both methods.

IV. Conclusion

A modal method for the efficient analysis of microwave waveguide filters based on multiple discontinuities has been developed and assessed. Its performance shows that the method is more efficient and accurate compared to numerical methods commercial software. This makes the new approach very competitive for its usage in the automatic design of such devices.

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