Study of a set of symmetric temporal transformations for the study of the orbital motion

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The aim of this work is to achieve a set of transformations symmetric on the orbit that improves the study of the orbital motion. In the first place, we study a new anomaly defined as the mean between the true and antifocal anomalies. For this anomaly, we show that the most common quantities of the two-body problem can be obtained in closed form.

In order to apply this transformation to analytical theories of the perturbed two-body problem we obtain the analytical developments as Fourier series with respect to the new anomaly of the expressions used in the two-body problem, including the Kepler equation.

This family can be included in the bi-parametric family of transformations defined by $d\Psi_\alpha = C_\alpha r^\alpha r'^{1-\alpha}$ where r is the vector radius of the secondary with respect to the primary and r' the vector radius with respect to the secondary focus of the ellipse. This family includes for $\alpha = 1$ the eccentric anomaly g, for $\alpha = 1/2$ the generalized lentgh of arc introduced by Brumberg and for $\alpha = 3/2$ the elliptic anomaly ω defined by Brumberg and Fukushima. This study is the natural extension of a set of previous works [6], [1], [2], [3], [4],[5] for the case of the symmetric anomalies.

Our study also involves the study the anomalies Ψ_α, providing the analytical developments of the main quantities of the two-body problem for an arbitrary

value of α in this family. This study is necessary to construct analytical theories of the orbital motion.

We are also concerned about the improvement in the numerical integration of the two-bodies problem provided by the use of the mentioned family of anomalies. For this purpose, we compute the values of α that optimize the process for each value e of the eccentricity. To test the robustness of the method, we consider a disturbed problem using as variable of integration for each step the anomaly Ψ_α for the optimal value α.

In the first place, the main contribution of this paper is the definition of a new anomaly symmetric on the ellipse, the study from an analytical point of view of the two-body problem using this new anomaly as temporal variable and the study of the Fourier series developments. In the second place we include this anomaly in a one-parametric family and we study the motion of two body problem with respect to an arbitrary anomaly in this family. In this case, we can generally obtain only the main quantities of the two-body problem as Fourier series of Ψ_α . To manage these expansions it is necessary to use a specialized software called Poisson Processor Series (PSP).

On the other hand, in order to study the local and global errors in the numerical integration we perform several numerical tests in order to obtain the values of the parameter that minimize the numerical integration errors, the dependence of the local truncation errors on the parameter values and the robustness of the methods using a perturbed problem as a test.

The main novelty of the method is the definition of a temporal variable through which it is possible to represent the problem of two bodies in closed form. This new variable can be included in a family that contains several classical anomalies. This family is studied in this work in order to obtain the analytical developments of the two-body problem and also to improve the performace of the numerical methods.

References

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