Improving the starting points for Newton's method under a center Lipschitz condition

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Keywords: Newton's method, semilocal convergence, error estimates, region of accessibility, restricted domains,

Abstract

Many problems from applied sciences can be brought in the form of an equation $F(x) = 0$, where F is a nonlinear operator defined on a nonempty open convex subset Ω of a Banach space X with values in a Banach space Y.

It is well-known that the solutions of these equations can be found in closed form only in special cases, that is why the solution methods for these equations are iterative.

For this, starting from one initial approximation x_0 of a solution x^* of the equation $F(x) = 0$, a sequence $\{x_n\}$ of approximations is constructed such that $\|x_{n+1} - x^*\| < \|x_n - x^*\|$, $n \geq 0$, that leds to the sequence $\{x_n\}$ converges to the solution x^*.

The study about convergence matter of iterative procedures is usually centered on two types: semilocal and local convergence analysis. In this study we are only concerned with the semilocal convergence matter which consists on, based on the information around an initial point, to give criteria ensuring the convergence of iterative procedure.

It is well-known that Newton's method,

$$x_0 \in \Omega, \qquad x_{n+1} = x_n - [F'(x_n)]^{-1}F(x_n), \quad n \geq 0, \tag{1}$$

is the most studied iterative method to approximate a solution x^*.

In this study motivated by previous results [1, 2, 3, 4, 5] and the idea of the restricted domains we analyse the semilocal convergence of the method from conditions on the starting point x_0 and the operator F and we improve the domain of starting points that converge to the solution.

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