Dynamical study of an optimal family of order four with applications

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Abstract

In this study we are concerned with the problem of approximating a locally unique solution x^* of equation

$$F(x) = 0, \tag{1}$$

where F is a differentiable function defined on a convex subset D of $\mathbb{C}$.

Many problems from Applied Sciences including engineering can be solved by means of finding the solutions of equations in a form like (1) using Mathematical Modelling. For example, dynamic systems are mathematically modeled by difference or differential equations, and their solutions usually represent the states of the systems. Except in special cases, the solutions of these equations can be found in closed form. This is the main reason why the most commonly used solution methods are usually iterative.

The dynamical properties related to an iterative method applied to polynomials give important information about its stability and reliability. In recently studies, authors such us Cordero et al. [2, 4], Amat et al [1], Chun et al. [3], Magreñán [5, 6], and others [7] have found interesting dynamical planes, including periodical behavior and others anomalies. One of our main interests in this paper is the study of the parameter spaces associated to a family of iterative methods, which allow us to distinguish between the good and bad methods in terms of its numerical properties.

In this work, we study the dynamical properties of the uniparametric family of iterative methods of order four, constructed by Biazar and Ghanbari

$$y_n = x_n - \frac{f(x_n)}{fx_n)}, \quad x_{n+1} = y_n - \frac{f(x_n)^2}{f(x_n)^2 - 2f(x_n)f(y_n) + 2\beta f(y_n)^2}\frac{f(y_n)}{f'(x_n)}$$

The results can be applied in many engineering areas, for instance, on calculating the Geodesic coordinates as a function of geocentric Cartesian coordinates.

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