On the solution of different eigenvalue problems associated with the neutron transport using the finite element method

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High-fidelity models of nuclear systems are done using the Boltzmann neutron transport equation [1], which in the approximation of G groups of energy is

$$\left(\vec{\Omega}\vec{\nabla} + \sigma_g\left(\vec{r}\right)\right)\psi_g\left(\vec{r},\vec{\Omega}\right) = \sum_{g'=1}^{G}\int_{\Omega} d\Omega'\sigma_{s,gg'}\left(\vec{r},\vec{\Omega}'\cdot\vec{\Omega}\right)\psi_g\left(\vec{r},\vec{\Omega}'\right) + \frac{1}{\lambda}\chi_g\sum_{g'=1}^{G}\nu\sigma_{f,gg'}\int_{\Omega} d\vec{\Omega}'\psi_{g'}\left(\vec{r},\vec{\Omega}'\right), \quad (1)$$

that is an integro-differential eigenvalue problem. In this work, to discretize the neutron transport equation the discrete ordinates method (S_N) is chosen and a high-order discontinuous Galerkin finite element method (DG-FEM) is

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used for the spatial discretization. After the discretization, a large algebraic generalised eigenvalue problem with rank deficient matrices is obtained.

We study the advantage of using Krylov subspace methods for these generalised eigenproblems as the Krylov-Schur method. This method is compared to the use of standard solvers like the Power Iteration method [3]. Krylov methods not only find the largest eigenvalue, used for criticality measurement, but also several subcritical eigenvalues are extracted with little additional computational cost. These subcritical eigenvalues and their corresponding eigenvectors can be used for time dependent modal methods. Also, the acceleration of the eigenvalue problem through the definition of energy-independent eigenvector is studied [2].

Numerical results are presented for some challenging one-dimensional and two-dimensional benchmarks. The results indicate that the Krylov-Schur method is an efficient technique to solve the eigenvalue problems arising from the steady state transport neutron equation.

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