PROPOSAL OF A GRAPHIC MODEL FOR SOLVING DELAY TIME MODEL INSPECTION CASES OF REPAIRABLE MACHINERY

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1. INTRODUCTION

The maintenance of railway fleets is a problem of great relevance at present. An optimum maintenance has significant implications in society, both in terms of safety and economics.

The application of innovative methods and technologies to the railway maintenance has shown in real cases increases in productivity and service, with savings up to 23%, and investments that can be amortized in less than two years (González-Fernández, 2012).

The implementation of requirements for the calculation of the Life Cycle Cost (LCC) has driven the development of railway maintenance. In this regard, the European Standard UNE-EN 60300-3 defines the calculation of the LCC as an economic analysis process to determine the life cycle cost of the product during its life cycle or a part thereof. The life cycle is considered as the time interval between the conception of the product and its elimination, and the LCC is the accumulated cost of the product during its life cycle. The LCC procedure provides customer service and a clear competitive advantage (Dunk, 2004).

This paper focuses on the study of the optimization of the costs of railway maintenance under the philosophy of LCC, studying the main parameters of optimization of the maintenance plan from the perspective of the reliability engineering. For doing this, the Delay Time Model approach is adapted to the railway case and a proposal of graphic resolution method is presented.

2. CONCEPT OF DELAY TIME MODEL

The basic Delay Time Model for a complex system was developed by Christer and Waller (1984). It is based on the concept of the delay time *h* of a defect, which is the time that elapses since a defect can be detected by means of an inspection until the defect becomes a fault. The process is divided into two stages (see Figure 1), allowing the development of different delay time models for maintenance policy optimization (e.g. Wang (2002), Jardine et al. (2006)).

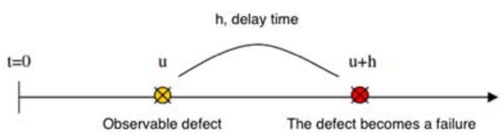

Figure 1. Time relationship between observable defect, failure and delay time

The main difficulty in this family of models lies in the determination of the initial time distributions *u* and the delay time *h*. This paper proposes an adaptation of the Delay Time Model by using graphic methods. This type of optimization method based on inspection with revealed failures is considered adequate for the optimization of the rail safety inspections.

Given *u* and *h* distributions, the inspection policy to be modelled is characterized according to the following hypotheses (H):

- H1. Inspections are carried out every *T* units of time with a cost *Ci* (currency unit), duration *ti*, being $T >> t_i$.

- H2. Inspections are perfect: (a) in case of any defect, it is detected; (b) the inspection does not contribute to degrade the system.
- H3. In case of any defect detection during the inspection, it is repaired in *ti* at a *CCM* cost within the inspection period. It is considered enough resources to address repairs.
- H4. The repair is perfect (the component of the repaired system stays in the initial condition).
- H5. Defects follows a Homogenous Poisson Process with a rate of defects occurrence, λ, constant.
- H6. In case of failure, it is immediately repaired at a time *dp* with a cost C_b, which comprises the repair cost and a penalty cost.
- H7. The repair time complies $d_p << T$.
- H8. The delay time *H* of a random defect is described by a probability density function *f(h)* and a cumulative distribution function *F(H)*, independent of the initial point of occurrence of the defect U.
- H9. $C_b > C_{CM}$.
- H10. The probability density function of the delay time, *f(h),* is known.

3. GRAPHIC MODEL PROPOSAL FOR THE DELAY TIME MODEL

It is selected a Delay Time Model formulation with exponential distributions for the occurrence of defects *u(t)* and the delay time *h(t).* The study considers a period with a single inspection and N defects. A number *B* of defects will not be detected, implying a penalty. The model is expressed as

$$C(T) = \frac{B(T) * C_b + C_{CM}\left(N(T) - B(T)\right) + C_i}{T + d_i} \tag{1}$$

where *C(T)* is the total cost per unit of time in the cycle; C_b are the penalty costs, C_{CM} are the corrective maintenance costs, and C_i are the inspection costs ($C_b > C_{CM} > C_i$); *T* is the time of the periodic inspection cycle; *di* is the time required for a periodic inspection; *N* represents the total number of defects in a cycle. The defect detection rate is considered to follow an exponential distribution; the total number of failures in the period is given by

$$N(T) = \lambda * T \tag{2}$$

Considering an exponential distribution for the delay time, the number of failures per cycle *B(T)* is expressed as

$$B(T) = \int_0^T \lambda * F(T-u)du = \int_0^T \lambda * (1 - e^{-\beta(T-u)})du = \lambda * \left(\frac{e^{-\beta T} - 1}{\beta} + T\right) \tag{3}$$

Substituting the terms of Eq. (1), simplifying and taking into account that T>>di (in railway maintenance, safety inspections usually last minutes and inspection periods, months), it is obtained the following expression

$$C(T) = \frac{\lambda * \left(\frac{e^{-\beta T} - 1}{\beta} + T\right) * C_b - \lambda * \left(\frac{e^{-\beta T} - 1}{\beta}\right) * C_{CM} + C_i}{T} \tag{4}$$

This equation seeks to obtain the total cost per unit of time for a programmed inspection period. It can be observed that the result depends on three cost variables (C_i, C_b, C_{CM}) and two statistical variables (λ, β). The multiple dependence of the equation in this general form decreases its application usefulness, since in practice it is not intuitive understanding the variations of the final result in function of the precision of each of the available variables. This research intends to understand the weighting of each variable, while developing a simplified method for estimating the minimum cost inspection cycle. For doing this, certain algebraic manipulations are carried out.

If Eq. (4) is reordered by grouping terms according to their dependence on the inspection period, the following expression is obtained:

$$C(T)=\frac{\lambda*\left(\frac{e^{-\beta T}-1}{\beta*T}+1\right)*C_b*T-\lambda*\left(\frac{e^{-\beta T}-1}{\beta*T}\right)*C_{CM}*T+C_i}{T}=\lambda*C_b+\frac{C_i}{T}+\lambda*(C_b-C_{CM})*\left(\frac{e^{-\beta T}-1}{\beta*T}\right) \quad (5)$$

$$C(T)=\lambda*C_b+\frac{C_i}{T}+\lambda*(C_b-C_{CM})*\left(\frac{e^{-\beta T}-1}{\beta*T}\right)=\lambda*C_b+C_i*\left(\frac{1}{T}+\lambda*\frac{(C_b-C_{CM})}{C_i}*\left(\frac{e^{-\beta T}-1}{\beta*T}\right)\right) \quad (6)$$

The corrected rate of failure is defined as

$$\Lambda=\lambda*\frac{(C_b-C_{CM})}{C_i}=\lambda*K \quad (7)$$

Owing $C_b > C_{CM} > C_i$ and $C_b > C_{CM} + C_i$, the cost ratio (K) is greater than 1 (Eq. (8)). These relationships must be fulfilled to make sense the program of inspections. Otherwise it would be carried out a replacement program based on schedule or age.

$$K=\frac{(C_b-C_{CM})}{C_i}>1 \quad (8)$$

Thus, the corrected rate of failure (Λ) will be the failure rate of the equipment weighted by a factor that will usually raise its order of magnitude by one or two orders in the railway case. Substituting Λ into Eq. (6) it is obtained the following expression

$$C(T)=\lambda*C_b+C_i*\left(\frac{1}{T}+\Lambda*\left(\frac{e^{-\beta T}-1}{\beta*T}\right)\right) \quad (9)$$

Observing Eq. (9) it is deduced the cycle cost is obtained as a sum of two terms:

(i) $\lambda*C_b$, is the cost of a cycle in which no inspection is carried out. It represents the maximum cost, calculated as the number of failures in cycle λ multiplied by the cost that would imply all the failures be penalizable, C_b. This term does not depend on the inspection period and will be denominated as the maximum cost of cycle, C_{max}.

(ii) The second term $C_i*\left(\frac{1}{T}+\Lambda*\left(\frac{e^{-\beta T}-1}{\beta*T}\right)\right)$ represents the achievable savings through a policy of periodic inspections. It depends on the inspection period and is the term to be optimized.

Eq. (9) can also be expressed as (Eq. (10), Figure 2).

$$C(T)=C_{\max}-A(T) \quad (10)$$

where $C_{\max}=\lambda*C_b$, and $A(T)=-C_i*\left(\frac{1}{T}+\Lambda*\left(\frac{e^{-\beta T}-1}{\beta*T}\right)\right)$

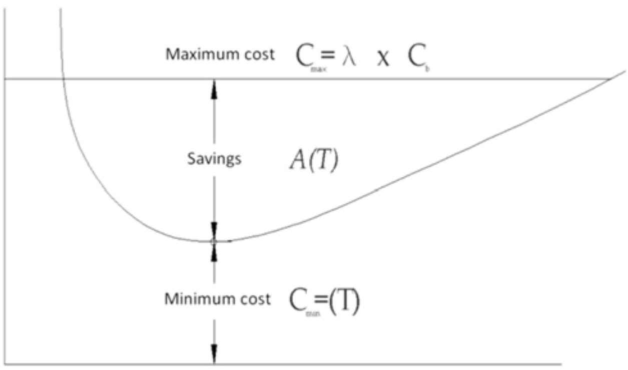

Figure 2. Costs per cycle based on the inspection period

4. USEFULNESS OF THE MODEL

Eq. (10) shows once the transformation of the starting model (Eq. 1) has been carried out, the saving term A(T) is the only one dependent on the inspection period. In addition, the number of variables of the problem has been reduced from the initial five, to three: period, delay factor (β) and corrected rate of failure (Λ).

The utility of the graphic method presented (Figure 3) is the representation of the optimum period in which saving is maximum. In order to graph the results, it has only be considered the term which varies with the period, the function $\frac{A(T)}{-C_i} = -C_i * \left(\frac{1}{T} + \Lambda * \left(\frac{e^{-\beta T} - 1}{\beta * T}\right)\right)$

For doing this, it is intended to solve the equation in two domains, time and cost, according to the following objectives:

1. Obtain the optimal inspection period to maximize the saving function based on β and Λ;
2. Obtain the maximum achievable saving Ao for this optimal inspection period, based on β and Λ.

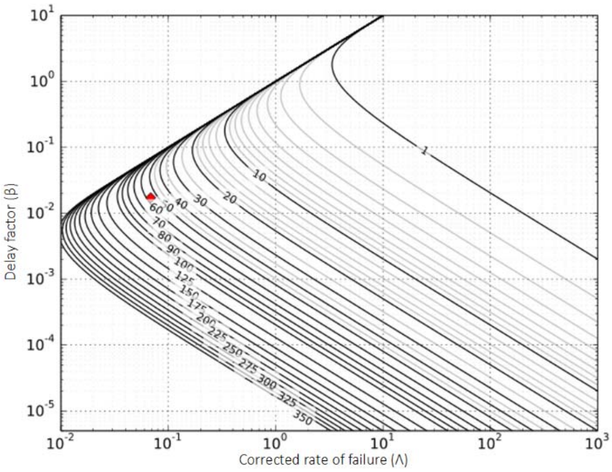

Figure 3. Optimum inspection period to obtain maximum savings per cycle based on Λ and β (T> ti)

The need for an easy graphic representation of the model has involved a mathematical transformation that will be presented in the full version of the paper.

5. CONCLUSION

This paper presents a new model of the Delay Time Model with exponential distributions for its application in the railway maintenance. The objective is to speed up the calculation for its practical application in this type of operations. With this graphic model presented, a mathematical model can be applied for the calculation of the delay time in tasks of rail transport equipment and fleets maintenance in an agile and precise way, being this is the main usefulness of the model.

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