

Valuation fuzzy soft sets: a fuzzy soft set based decision making approach to valuation of goods*

José Carlos Rodríguez Alcantud[†] Salvador Cruz Rambaud[‡]

María José Muñoz Torrecillas[§]

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Abstract

This paper introduces partial valuation fuzzy soft sets and presents a flexible methodology in order to implement this concept. We take advantage of such design in order to value goods, both theoretically and through a fictitious example. We aim to apply this valuation methodology, based on fuzzy soft sets, to a real case study in the field of real state valuation.

Key-words: Fuzzy soft set, linear regression, valuation of goods, data filling, decision making.

1 Soft sets and fuzzy soft sets: notation and definitions

Let X denote a set. Then $\mathcal{P}(X)$ is the set of all non-empty subsets of X . A fuzzy subset (also, FS) A of X is a function $\mu_A : X \rightarrow [0, 1]$. For each $x \in X$, $\mu_A(x) \in [0, 1]$ is the degree of membership of x in that subset. The set of all fuzzy subsets of X will be denoted by $\mathbf{FS}(X)$.

In soft set theory, we refer to a universe of objects U , and to a universal set of parameters E .

Definition 1 [Molodtsov [7]]. Let A be a subset of E . The pair (F, A) is a *soft set* over U if $F : A \rightarrow \mathcal{P}(U)$.

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[†]BORDA Research Unit and Multidisciplinary Institute of Enterprise (IME), Universidad de Salamanca, 37007 Salamanca, Spain

[‡]Departamento de Economía y Empresa. Universidad de Almería (Spain). Phone: +34 950 015 184. Fax: +34 950 015 178. E-mail: scruz@ual.es.

[§]Departamento de Economía y Empresa. Universidad de Almería (Spain). Phone: +34 950 214 135. Fax: +34 950 015 178. E-mail: mjmtorre@ual.es.

The pair (F, A) in Definition 1 is a parameterized family of subsets of U , and A represents the parameters. Then for every parameter $e \in A$, we interpret that $F(e)$ is the subset of U approximated by e , also called the set of e -approximate elements of the soft set.

Other interesting investigations expanded the knowledge about soft sets. Maji, Bismas and Roy [6] defined the notions of soft equalities, intersections and unions of soft sets and soft subsets and supersets. Feng and Li [4] studied various types of soft subsets and soft equal relations. Soft set based decision making was initiated by Maji, Biswas and Roy [5], and subsequently further applications of soft sets in decision making contexts were given [3, 8, 9].

The set of all fuzzy soft sets over U will be denoted as $\mathcal{FS}(U)$. Any soft set can be considered as a fuzzy soft set with the natural identification of subsets of U with FSs of U (cf. Alcantud [2]). If for example, our universe of options are films which are parameterized by attributes, then fuzzy soft sets permit to deal with properties like “funny” or “scary” for which partial memberships are almost compulsory. However soft sets are suitable only when properties are categorical, e.g., “Oscar awarded”, “3D version available”, or “silent movie”.

In real practice both U and A use to be finite. Then k and n will denote the respective number of elements of U and A . In such case soft sets can be represented either by $k \times n$ matrices or in their tabular form (cf. Alcantud [1]). The k rows are associated with the objects, and the n columns are associated with the parameters. Both practical representations are binary, that is to say, all cells are either 0 or 1. One can proceed in a similar way in fuzzy soft sets, but now the possible values in the cells lie in $[0, 1]$.

2 Some novel concepts related to valuation fuzzy soft sets

In this Section we introduce the main new notions in this paper, namely, valuation and partial valuation fuzzy soft sets. In order to define our novel notions we refer to a universe U of k objects, and to a universal set of parameters E .

Definition 2. Let A be a subset of E . The triple (F, A, V) is a *valuation fuzzy soft set* over U when (F, A) is a fuzzy soft set over U and $V \in \mathbb{R}^k$. We abbreviate valuation fuzzy soft set by VFSS.

The set of all valuation fuzzy soft sets over U will be denoted as $\mathbf{V}(U)$. If we restrict Definition 2 to soft sets over U , a particular concept of *valuation soft set* is naturally produced. In these notions the idea is that each option from U is associated with a valuation, appraisal or assessment, in addition to the standard parameterization of U as a function of the attributes in A . For example, in the usual example where the options are houses this valuation may be the market price. But it can also be defined from elements from fuzzy soft set theory. We proceed to formalize these ideas.

Definition 3. A *rating procedure* for fuzzy soft sets with attributes A on a universe U is a mapping

$$\Pi : \mathcal{FS}(U) \longrightarrow \mathbf{V}(U).$$

Any rating procedure associates every FSS over U with a VFSS over U . In particular, we can use scores associated with decision making mechanisms from the literature (e.g., fuzzy choice values, Roy and Maji [10], Alcantud [1]).

Definition 4. Let A be a subset of E . The triple (F, A, V^*) is a *partial valuation fuzzy soft set* over U when (F, A) is a fuzzy soft set over U and $V^* \in (\mathbb{R} \cup \{*\})^k$. We abbreviate partial valuation fuzzy soft set by PVFSS.

The set of all valuation fuzzy soft sets over U will be denoted as $\mathbf{V}^*(U)$. If we restrict Definition 4 to soft sets over U , we define the particular concept of *partial valuation soft set*. As in VFSSs, each option from U is associated with a valuation in addition to the standard parameterization of U as a function of the attributes in A . But in PVFSSs some of the valuations may be missing. We represent such missing information by the $*$ symbol.

3 Data filling in PVFSSs

Valuation is an abstract concept. It may be specialized in many ways. We take advantage of this issue in order to fill missing data in PVFSSs. The idea is: we use a rating procedure and then as long as there are 2 values in V^* that belong to \mathbb{R} , we apply a regression equation to fill the missing data. Our rating procedure is defined as follows.

Definition 5. Alcantud's rating procedure is defined by the expression $\Pi_a(F, A) = (F, A, V^{\Pi_a} = (\Pi_a^1, \dots, \Pi_a^k))$, where $\Pi_a^i = S_i$ is the score associated with option i .

We may use regression models other than linear regression. Then we need more number of non-missing data.

4 Valuation of goods: proposal and an example

We proceed to define our class of procedures for the valuation of goods when (a) these goods are characterized by parameters, and (b) there are comparable goods that are characterized by the same parameters. In other words, we have information about the goods in the form of PVFSSs. This is discussed in Section 4.1. Then in Section 4.2 we pose a practical example that illustrates the implementation of our approach.

4.1 A new procedure for valuation of goods

Given a list of options characterized by a PVFSS, we may use the information on the known values in order to fill the missing data. By doing so we are

valuating the goods with missing values. Therefore we can use the data filling procedure described in Section 3 in order to make decisions e.g., as to which prizes should be attached to properties which are put into the market.

4.2 An example of valuation of properties

Table 1 represents a PVFSS. It uses the input data of Table 6 in Alcantud [1], which is then complemented with some partial valuations of the alternatives.

| | p_1 | p_2 | p_3 | p_4 | p_5 | p_6 | p_7 | V_i |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| o_1 | 0.036 | 0.015 | 0.064 | 0.216 | 0.048 | 0.054 | 0.405 | 137 |
| o_2 | 0.144 | 0.084 | 0.360 | 0.045 | 0.036 | 0.020 | 0.175 | 109 |
| o_3 | 0.120 | 0.084 | 0.180 | 0.030 | 0.096 | 0.021 | 0.294 | 97 |
| o_4 | 0.504 | 0.192 | 0.108 | 0.006 | 0.048 | 0.048 | 0.096 | * |
| o_5 | 0.084 | 0.245 | 0.036 | 0.096 | 0.270 | 0.200 | 0.140 | 192 |
| o_6 | 0.216 | 0.315 | 0.042 | 0.108 | 0.224 | 0.126 | 0.135 | 198 |

Table 1: Tabular representation of the partial-valuation fuzzy soft set (S, P) in Section 4.2. All V_i 's are expressed in thousands of euros.

We are interested in selling property o_4 , whose market value we want to assess. The options o_i include our property and other real state properties for sale, and they are all characterized by the p_j attributes. An inspection of the market shows that recent purchases in the same area or street amounted to the respective V_i 's. By selecting the rating procedure Π_a then

$$V^{\Pi_a} = (-1.3, -3.2, -3.78, -2.24, 5.24, 5.26)$$

because Π_a values the options by Alcantud's scores which are given in [1, Table 8]. It is immediate to define $\mathcal{V} = (-1.3, -3.2, -3.78, -2.24, 5.24)$ and $I = \{1, 2, 3, 5, 6\}$.

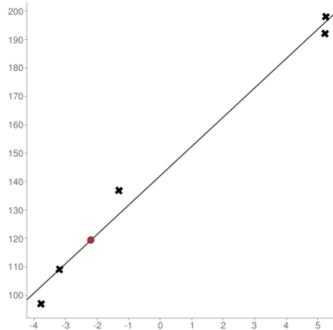


Figure 1: The regression line in Section 4.2. The red dot shows the valuation of the missing option o_4 , with score -2.24 at the horizontal axis.

We now need to calculate the linear regression equation from the bivariate data that combine the known valuations with the corresponding component of our rating procedure, which is

$$((-1.3, 137), (-3.2, 109), (-3.78, 97), (5.24, 192), (5.26, 198))$$

Some easy computations show that the regression line equation is

$$y = 142.02204039129 + 10.310719839438x$$

and then if we input $x = -2.24$ we obtain 118.92602795094888. This means that option o_4 should be valued by 118.926, 03 euros.

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