Effects of random environment and delayed responses on the dynamics of a semiarid vegetation model

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Abstract

In this work, we consider a simplified model of semiarid vegetation and investigate the effects on the dynamics of the system of considering the existence of lagged responses and random environmental parameters.

It has been described, from experimental as well as modelling approaches, that vegetation dynamics in semiarid ecosystems may present complex dynamics, with hysteresis in degradation and recovery trajectories, and alternative stable states that may led to critical transitions to irreversible degraded states, the so called catastrophic shifts. Temporal delays and environmental noise have been recently pointed out as potential factors affecting the dynamics of different population models [1, 2] Delay differential equations [3] and random differential equations [4] have been considered in different models of population dynamics (e.g., [5, 6]).

The base model considered in this work is a simplified version of the mean field approximation to a spatially explicit cellular automata model described by Kéfi et al. [7]. The original model in [7], as well as its mean field approximation, considered three different states: vegetated, empty, and degraded, and presented alternative stable states depending on a bifurcation parameter representing the probability of vegetation colonization. This model included competiton and facilitation effects, and has been used to describe potential

early warning indicators of the possibility of catastrophic shifts [8], and to incorporate hydrology mediated resource redistribution [9].

The simplified mean field model considered in this work consist of only two states, vegetated and empty, the probability of a cell being in the vegetated state, $x(t)$, satisfying the equation

$$x'(t) = x(t)(b - cx(t))(1 - x(t)) - mx(t), \tag{1}$$

where m is the mortality rate, and b and c are parameters representing colonization and competition effects.

We consider first the dynamics of this simplified model, which does not present alternative stable states, and then investigate how that dynamics is affected by including delayed responses. We also consider the effects, on the base model (1) and the model with delayed effects, of considering environmental noise, by including parameters as random variables. Numerical experiments illustrating the results are presented.

Novelty of the approach and mathematical methods

Investigations on the joint effects of delayed responses and random environment on vegetation models are very limited, and restricted to very simple population models with Gaussian noise. The analysis carried out in this work, based on a well known semiarid vegetation model, is completely new.

Mathematical methods used in this work include, among others, computation of numerical solutions of the different models considered, including random delay differential model, and the analysis of their dynamics.

References

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