Modeling of biological control of plant viruses with seasonality

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Abstract

Man cannot survive without plants. Man eats both plants and animals that feed on plants. Plants also are sources of medicines, fibers for clothes, even building materials and are essential for a healthy environment and for life in general. Just like humans and animals, plants are subject to a variety of diseases many of which are caused by viruses. Some viruses cause only temporary damage to the plants but others will actually kill them. As a result, billions of dollars are lost every year because of virus related crop loss. Most of the time, virus propagation from plant to plant is done by an insect vector, usually insects that bite infected plants and then bite susceptible plants. Insect vectors typically have a seasonal behavior. They are very active in the warm months and not very active, almost dormant, in the cool months. To combat the vectors, chemical insecticides are commonly used as a control. Unfortunately, these chemicals have toxic effects on humans, animals and even the plants and the environment in general. An alternative is to introduce a predator, or just increase the number of a naturally present one, to prey on the insects. This will ultimately control the vector population. In this paper we first consider a model of the interactions between the vectors, the plants and the predators with seasonal infection rates and later delays will be introduced.

In the mathematical model introduced, there are six populations: susceptible, infected and recovered plants, susceptible and infected vectors, and predators. A susceptible plant can become infected, if an infected insect feeds on it and is able to transmit the virus to the plant. The infected plant will either die from the virus or recover. A healthy vector will can obtain the virus by feeding on an infected plant. The infected vectors have no ill effects from the virus so they do not fight the virus and therefore they do not recover. The virus does not affect the predators.

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The total number of plants is assumed to be constant since the framers will replace a dead plant with a healthy one. This assumption allows us to eliminate the recovered plant population from the model since it can be calculated from the total constant plant population.

The first model we construct is based on a system of nonautonomous ordinary differential equations. The equations are nonautonomous because the coefficients dictating the contact rates between plants and insects are time dependent, in particular we will take them periodic, due to the seasonality of the vectors. The system may have periodic and steady solutions. Of particular interest is the stability of the disease free equilibrium point, since we want to find conditions for the disease to disappear. To analyze this stability, the basic reproductive number R_0 is a very useful concept. It is the number of secondary infections an infected individual generates. If $R_0 < 1$, the disease will die out as $t \to \infty$, but if $R_0 > 1$, a time dependent endemic will occur. For autonomous systems, the basic reproductive number is constant. However, for nonautonomous systems, R_0 is a function that depends on time. Thus, R_0 can be less than 1 for some values of t and greater than 1 for others. So the question is what happens to the stability of the disease free equilibrium. After calculating R_0, some numerical simulations for different cases will be given to demonstrate the analytical results.

The mathematical model will then be extended to include Holling type 2 functional response for the susceptible insects, infected insects, and predators. Moreover, predators will be added to the system at a constant rate. Calculating R_0 analytically is difficult for this model, so numerical approximations will be made. Numerical simulations will then be used to approximate the solutions for the system. Finally, the model will be improved by introducing two time delays. The introduction of these two delays will convert the system of ordinary differential equations to a system of delay differential equations. The two delays considered are: the time it takes from the moment a plant is bitten by an infected vector to the moment the plant is infected; and a second smaller delay for the time from the moment a vector bites an infected plant to the moment the vector becomes infected. The system of delay differential equations needs to be studied numerically and comparisons with the non delay system will be presented.

The mathematical model of virus propagation with seasonality in plants with a predator and including delays is new. From the practical side, the model can be used to determine the amount of introduced predators necessary to eliminate the plant disease under different conditions.

keywords: delay differential equations, plant viruses, mathematical modeling, seasonality