Some awkward issues on pairwise comparison matrices

Julio Benítez[1], Silvia Carpitella[1,2], Antonella Certa[2], Amilkar Ilaya-Ayza[3], Joaquín Izquierdo[1]

[1]Instituto Matemático Multidisciplinar/Universitat Politècnica de València, València, Spain, {jbenitez,jizquier}@upv.es

[2]Dipartimento dell'Innovazione Industriale e Digitale (DIID), Ingegneria Chimica, Gestionale, Informatica, Meccanica/Università degli Studi di Palermo, Palermo, Italy, silvia.carpitella@unipa.it, antonella.certa@unipa.it

[3]Facultad Nacional de Ingeniería, Universidad Técnica de Oruro, Ciudad universitaria s/n, Oruro, Bolivia, amilkar.eia@gmail.com

Abstract

Many decision-making techniques use pairwise comparisons (PCs) elicited by one or more stakeholders involved in a given corresponding decision-making process. In very complex problems, the number of criteria or options to be compared may be too large, thus limiting PC applicability to large-scale decision problems due to the so-called curse of dimensionality, that is, a large number of pairwise comparisons need to be produced from a decision maker. For example, in AHP, Saaty [1] recommends that to obtain a reasonable and consistent PC matrix, the number of comparing elements should be at most seven. In [2] and [3], arguing on limitations of the human capabilities, the maximum number of elements to be handled simultaneously is approached by a maximum of seven plus two. This limit, also called 'channel capacity', a measure of our ability to process information, is widely known in the literature (see Miller [4]), and refers to the number of elements that can be held in short term memory simultaneously.

Miller also stated that his magic number was for items with one aspect or attribute, and reported this to be true for various tasks. The aim was to assess how well humans can separate various levels of intensity of certain particular stimuli. Among these stimuli we have frequency, position in an interval, loudness, saltiness, size, etc. However, when more attributes are considered, then one can remember more, depending on his or her familiarity with the situation and the complexity of the subject. However, according to [5], Miller's magical number seven is only true for information that has one memory attribute or function – and we can work with many more. Marnell [6], using one of the examples provided by Miller, states that 'there is... no parallel between naming tonal frequencies according to a provided legend and unravelling the meaning of a map, block...' (this last part of the comparison refers to more complex information structures). This same author also states that Miller's article discusses what he calls the span of immediate memory (also known as the capacity of our short-term memory), and makes it clear that 'the capacity of our short-term memory might well be relevant to our ability to take in and comprehend material at the atomic level ... but at the molecular level ... its relevance is doubtful', and that '... short-term memory is the very stuff of Miller's paper, especially its role in judgment, attention and recall'. Marnell claimed that this theory 'needs to be radically updated to bring it into line with current knowledge in cognitive psychology', and he cites [7] to conclude that 'a limit of 7 ± 2 is yesterday's guesstimate. Today it is 4 ± 1 for unrelated items and 15 for... [related concepts]'.

In this paper we argue that in special cases comparison matrices of more than the traditional 7 ± 2 elements may be valid. For example when a renowned expert on a subject is confronted with an high number of elements and there is no clear possibility of clustering them following some homogeneity criteria as suggested in [2]. This is the most salient feature of the case study we address in this paper: a team of experts working on a daily basis with the management and operation of a utility infrastructure were confronted with the problem of elucidating about the ease

of operation of fifteen individual parts of the infrastructure. The PC matrix was eventually built and taking into account the team profound expertise in the problem, the matrix was considered to encompass reliable information, according to our claim. Nonetheless, aware of the problems associated with dealing with so many elements, for example when working with alternatives or when including the PCs in an analytic network process (ANP) solution of the problem, and after observing certain structured patterns in the matrix, we thought of devising a method to consistently shrink or compress the comparison matrix, thus reducing the size of the problem. In this paper, using a linearization technique already introduced by the authors [8], we develop such a compressing or merging technique so that certain elements may be synthesized to produce a new comparison matrix that gathers some elements into clusters, while maintaining the experience and the perception of the experts, and also the consistency and, eventually, reducing the size of the problem, thus making it more manageable. Of course, the final approval of the panel of experts is deemed absolutely necessary.

This size reduction is devised to be useful for a number of technical purposes. For example, to avoid PCs when assessing alternatives with respect to the criteria; reciprocally, the number of comparisons will also be lower when providing PCs among the criteria for any of the considered alternatives (in the case the ANP methodology is used).

Regarding other problems, this comparison matrix clustering technique may also be useful in other interesting contexts, such as, for example, the merging of companies in stoke markets once an alliance has been produced to avoid the whole process of PCs from scratch.

References

[1] Saaty, T.L. A scaling method for priorities in hierarchical structures, Journal of Mathematical Psychology 15, 1977, 234-281.

[2] Saaty, T.L., Ozdemir, M.S. Why the Magic Number Seven Plus or Minus Two, Mathematical and Computer Modelling, 2003, vol. 38, pp. 233-244.

[3] Ozdemir, M.S. Validity and inconsistency in the analytic hierarchy process, Applied Mathematics and Computation, 2005, vol. 161, pp. 707-720.

[4] Miller, G.A. The Magic Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information, The Psychological Review, 1956, vol. 63, pp. 81-97.

[5] https://rapidbi.com/millermagicalnumberseven7/ (accessed on 05/04/2015)

[6] Marnell, G. Chunking the Information Presented to Readers. A critique of Information Mapping. The Australian and New Zealand Journal of Technical Communication, 33, October 2014 (http://www.abelard.com.au/information%20mapping.pdf).

[7] Baddeley, A. Working Memory, Thought, and Action, Oxford University Press, Oxford, 2007, p. 143.

[8] Benítez, J., Delgado-Galván, X., Izquierdo, J., Pérez-García, R. Achieving matrix consistency in AHP through linearization. Applied Mathematical Modelling 35 (2011) pp. 4449-4457.