Metamaterial Acoustics on the (2+1)D Schwarzschild Plane

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As early as 1916 Karl Schwarzschild found his simple—albeit nontrivial—spacetime solution of Einstein's equations for the gravitational field with underlying static and spherical symmetry [1]. In the beginning considered to be a mathematical curiosity and only of academic interest, it has now in the age of high-precision GPS navigation and black-hole astronomy become the center stage of many practical and important applications.

In $(n+1)$D spacetime geometry, the static and spherically symmetric metric of Schwarzschild-type takes the following general form

$$\boldsymbol{g} = -h(r)\, c^2 dt \otimes dt + h^{-1}(r) dr \otimes dr + r^2 d\Omega_{n-1} \otimes d\Omega_{n-1},$$

where t and r are the local time and radial coordinates. The solid angle Ω_n comprises the n angles of the n-sphere S^n. The radial function shall satisfy $h(r) > 0$ for all $r > 0$. The specific solution for $h(r)$ will depend on dimension n and the physical conditions imposed. For $n = 3$ the vacuum solution yields the conventional Schwarzschild metric. O'Neill [2] introduced the toy model with $n = 1$, which is conformally flat, and all solutions are automatically vacuum solutions. Here, we will propose the considerably richer case $n = 2$, a spacetime which we term the $(2+1)$*D Schwarzschild plane*, and will discuss its admissible solutions.

On the other hand, metamaterials recently have offered researchers and engineers unparalleled capabilities for the design and construction of artificial devices exhibiting remarkable properties far beyond those found in nature. Among this class of exceptional materials, acoustic metamaterials play an important rôle, see *e.g.* [3]. They allow to model acoustic phenomena with curved background spacetimes and make predictions for future laboratory experiments, which may also help to settle fundamental questions apart from the technical applications, see *e.g.* [4].

In this work we examine the possibility to implement acoustics on the $(2+1)$D Schwarzschild plane and study its wave propagation properties with conceivable asymptotic behaviour and event horizons. By using the differential-geometric framework developed in [5–7] we determine the corresponding acoustic parameters. Moreover, we derive the equations of motion which describe wave propagation for this spacetime. Finally, we present numerical estimates for some interesting examples.

Keywords: acoustic metamaterials, transformation acoustics, curved acoustic spaces, black holes, Riemannian manifolds, Schwarzschild metric.

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