

On Legendre Transformation for Hamiltonian Systems Corresponding to Second Order Lagrangians

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Abstract.

The paper is devoted to an extension of classical Hamilton–Cartan variational theory on fibered manifolds to the case of class second order Lagrangians. The aim of this paper is to announce some recent results in case of second order Lagrangians corresponding to 2nd and 3rd Euler–Lagrange form.

We are interested in second order Lagrangians which give rise to Euler–Lagrange equations of the 3rd order or “non-affine” 2nd order. All these Lagrangians are singular in the standard Hamilton–De Donder theory and do not have Legendre transformation. For such Lagrangian we find appropriate set of Lepagean equivalents (resp. family of Hamilton equations) whose admit a generalized Legendre transformation. We note that the generalized momenta p_{σ}^{ij} satisfy $p_{\sigma}^{ij} \neq p_{\sigma}^{ji}$.

The generalized Legendre transformation and its properties are illustrated on examples. We consider Hamiltonian systems for concrete above mentioned Lagrangians.

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