Numerical solution of the Burgers' equation by splitting methods using Crank-Nicolson schemes

M. Seydaoğlu[a,b] and S. Blanes*[b]

[a] *Department of Mathematics, Faculty of Art and Science, Muş Alparslan University, 49100 Muş, Turkey.*
[b] *Instituto de Matemática Multidisciplinar, Building 8G, second floor,, Universitat Politècnica de València. 46022 Valencia, Spain.*

Abstract

In this work, a second order symmetric splitting method and the Crank-Nicolson finite difference method are combined for solving the one-dimensional Burgers' equation

$$u_t = (\frac{u^2}{2})_x + \nu u_{xx}, \qquad u(x,0) = u_0(x).$$

numerically, where $\nu > 0$ is the kinematic viscosity parameter related to the Reynolds number $R = 1/\nu$, $x \in \mathbb{R}$ and $t \geq 0$, in one space dimension with Dirichlet boundary conditions. Most of numerical schemes proposed in the literature can not exhibit the correct physical behavior of the equations for very small values of viscosity. Therefore, the splitting method derived for near-integrable system have been employed. This method have positive real coefficients and can be used for non-reversible system such as Burgers' equation. However, we have used unconditionally stable Crank-Nicolson finite difference method to solve each simplified problem and the filtering technique to treat the nonlinear instability. We have compared the new numerical results with numerical and exact solutions reported in the literature and found that they are very accurate for small values of the viscosity.

*Corresponding author: serblaza@imm.upv.es