Magnus—Krylov methods for non-autonomous linear wave equations

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The work presented addresses the numerical integration of the second-order time-dependent linear partial differential equation

$$u_{tt}(x,t) = f(t,x)\,u(x,t), \qquad x \in \mathbb{R}^d,\ t \geq 0, \tag{1}$$

equipped with the initial conditions $u(x,0) = u_0(x)$ and $u_t(x,0) = u_0'(x)$.

After spatial discretization, (1) can be expressed as

$$y''(t) = M(t)y(t), \quad y(t_0) = y_0,\ y'(t_0) = y_0', \tag{2}$$

where $t \in \mathbb{R}$, $y \in \mathbb{C}^r$.

In general, the solution is oscillatory, and exponential integrators show a good performance since they, in turn, provide numerical solutions which reflect the oscillatory nature of the exact solution.

However, due to the large dimension of the problem, one has consider thoroughly the computational cost of the schemes. As $M(t)$ originates from the discretization of a PDE, methods that employ only matrix–vector products are computationally reasonable. Thus, action of the matrix exponentials in Magnus-decomposition methods is approximated by employing Krylov-type subspace methods.

We compared the solutions of non-autonomous problems obtained by the new methods, by their predecessors and by a set of well-known methods. The results show that Krylov-type exponential computation facilitates application of Magnus-decomposition methods, which were initially designed for solving matrix differential equations, to large-dimensional problems.

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